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Ontario
Water Resources
Commission

Great Lakes
Water Quality
Surveys Program

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GREAT LAKES NEARSHORE MODELLING FROM CURRENT METER DATA

1969

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ABSTRACT

Methods for the prediction of the dispersion patterns resulting from the continuous discharge of waste in the nearshore areas of lakes are developed. These methods are based upon the analysis of recording current meter records. The long-term dispersion characteristics are presented as monthly mean concentration contours for various discharges. The short-term characteristics are presented as five hour probabilities and dilution rates for the four major compass directions.

GREAT LAKES NEARSHORE MODELLING FROM CURRENT METER DATA

1969

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INTRODUCTION

The increasing density of both municipal and industrial water users on the shores of the Great Lakes can result in multiple user interference if the location of the water intakes and waste discharge are not judiciously selected. To be capable of objectively selecting the best locations, the Commission must have reliable mathematical models capable of predicting the nearshore processes. Unfortunately, the nearshore processes are complex in nature and modelling will necessarily be a slow and evolutionary process. The complexity of the problem is basically a result of the non-steady loading conditions, non-steady water transport, seasonal temperature stratification, the three-dimensional aspects of the problem and the local area topography. A typical example of the variation of the water transport is demonstrated in Figure 1 which portrays the water movement history at a point. The current measured in each hour is plotted as a straight line which shows the direction of the movement and the length of the line defines the magnitude. The lakeshore situation is similar in many respects to that encountered in coastal areas although the period of variation is not as regular as a tidal cycle and the stratification is mainly temperature generated and not salinity.

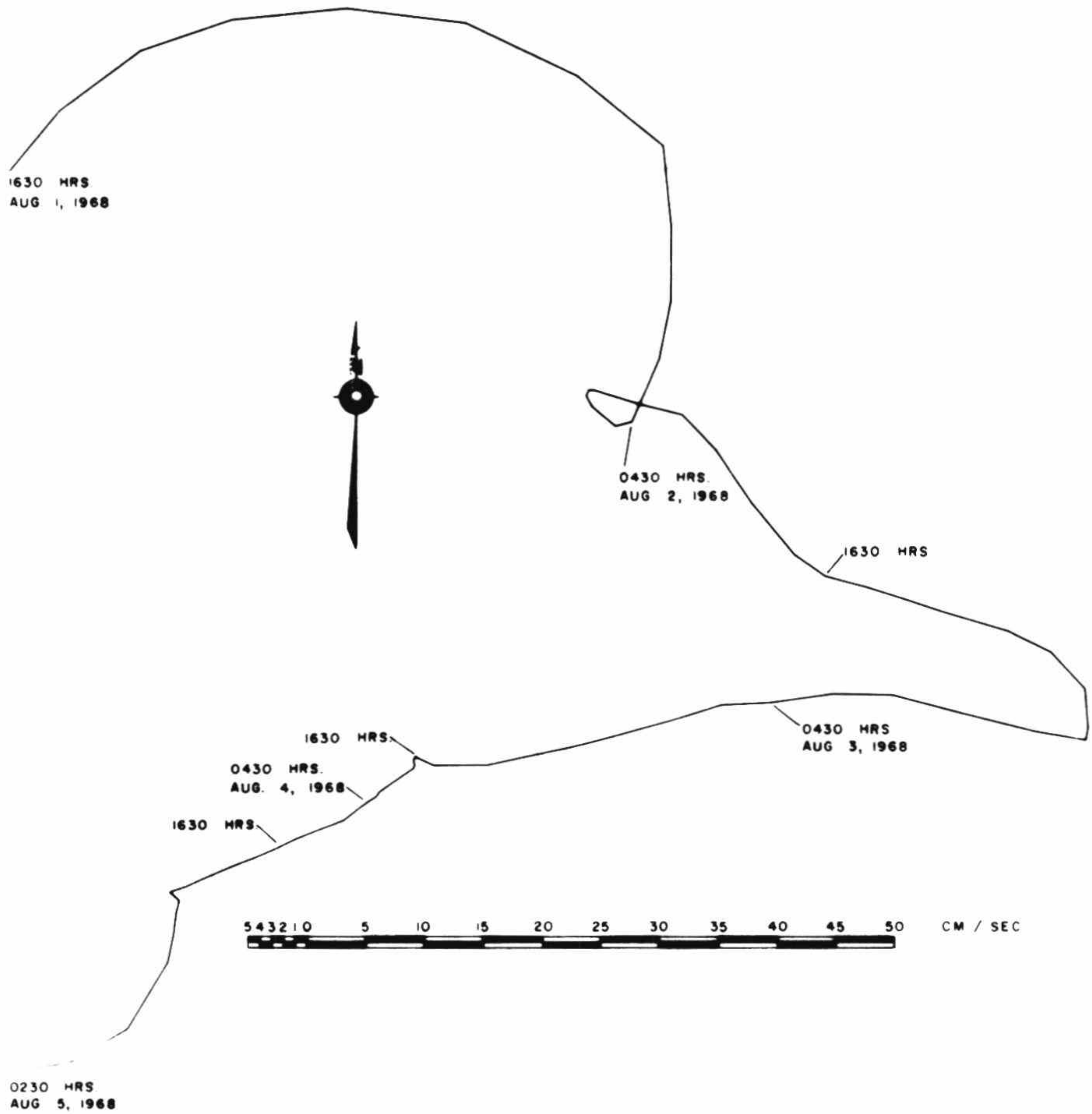

FIG. 1

NANTICOKE CURRENT METERS STUDIES - 1968

METER 022 - SUCCESSIVE HOURLY CURRENT VECTORS OCCURRING AT A POINT PLOTTED SEQUENTIALLY HEAD TO TAIL

The nature of both the water chemistry and currents has been found to be variable and significantly different on an hourly (Palmer (1), Hamblin (2), Verber (3)), daily (Brydges (4) and monthly basis (Palmer (5)). The complexity of the driving mechanisms and the different variations occurring over periods of hours, days and months, suggest that probably a systems approach of some kind is required to produce meaningful results. By systems approach, one means the gathering of large amounts of data history then trying to reproduce the results numerically. A systems approach has the disadvantage of not being deterministic. This means it is difficult to relate the results to a system of equations which define all the variables and how they relate to each other, e.g. force = mass x acceleration is a deterministic equation. However, it must be pointed out when the number of variables becomes larger and their interactions complex even the deterministic models will not reduce to simple expressions. The systems approach involves either a numerical difference technique or time series and probabilistic technique. The problem can be further simplified by considering different time scales separately, e.g. consider the hourly dispersion pattern or the monthly assimilation pattern or a yearly pattern. The latter pattern has already been extensively developed by the vessel monitoring survey program during the shipping season (April to November) depicting concentration contours of various parameters on the Great Lakes.

The shorter term patterns are to be discussed in the following text. The results are based upon collected recording current meter data which is subjected to time series and probabilistic analytical techniques. The methods are restricted to passive contaminants (substances which do not react with the water environment) which are merely transported and diluted by the water movements (e.g. chlorides, conductivity, etc.). The source considered here is a continuous steady release of waste in the nearshore areas. The method is essentially attempting to account for the water movement variations by holding the other variables (non-passive contaminants, non-steady loading conditions, seasonal temperature stratification and area topography) steady. The next phase of the development will incorporate more variables finally evolving into a more complete simulation or at least a more flexible model.

METHOD

General

The prediction of dispersion patterns in the Great Lakes in the past has been accomplished by employing classical techniques such as dye injection (Csanady (6), Palmer (7), Noble (8) and drogues Okubo (9), Hamblin (10)). These methods produce results based upon field measurements normally taken over a short period of time during daylight hours in

the summer on the Great Lakes. In some cases, it is necessary to assume some form of steady state conditions of currents or current patterns (currents constant in magnitude and direction for the study) to obtain the dispersion characteristics. Recording current meters installed in the Great Lakes have demonstrated the variability of the currents (Hamblin (2), Palmer (11), Verber (12)) over a 24-hour period as well as the differences existing in currents between different locations. The variability of the water chemistry at a point in the lake has also recently been shown to be significant (Brydges (4) and Palmer (5)). Generally, spectral analysis of currents produces significant peaks corresponding to the inertial period of the lake or a bay, geostrophic (earth rotational) periods and diurnal periods with their respective harmonics to name a few. Any determination of dispersion or assimilation characteristics must consider the variability recorded by the current meters.

The following two methods operate on a full month's data from a current meter considering the currents as a continuous function represented by a series of hourly averages. In other words, what happens in one hour is related to the previous hour and no sudden changes occur. As the methods are based upon records obtained at a point, it is necessary to determine how representative this point is of an area. This can be accomplished by operating another meter some

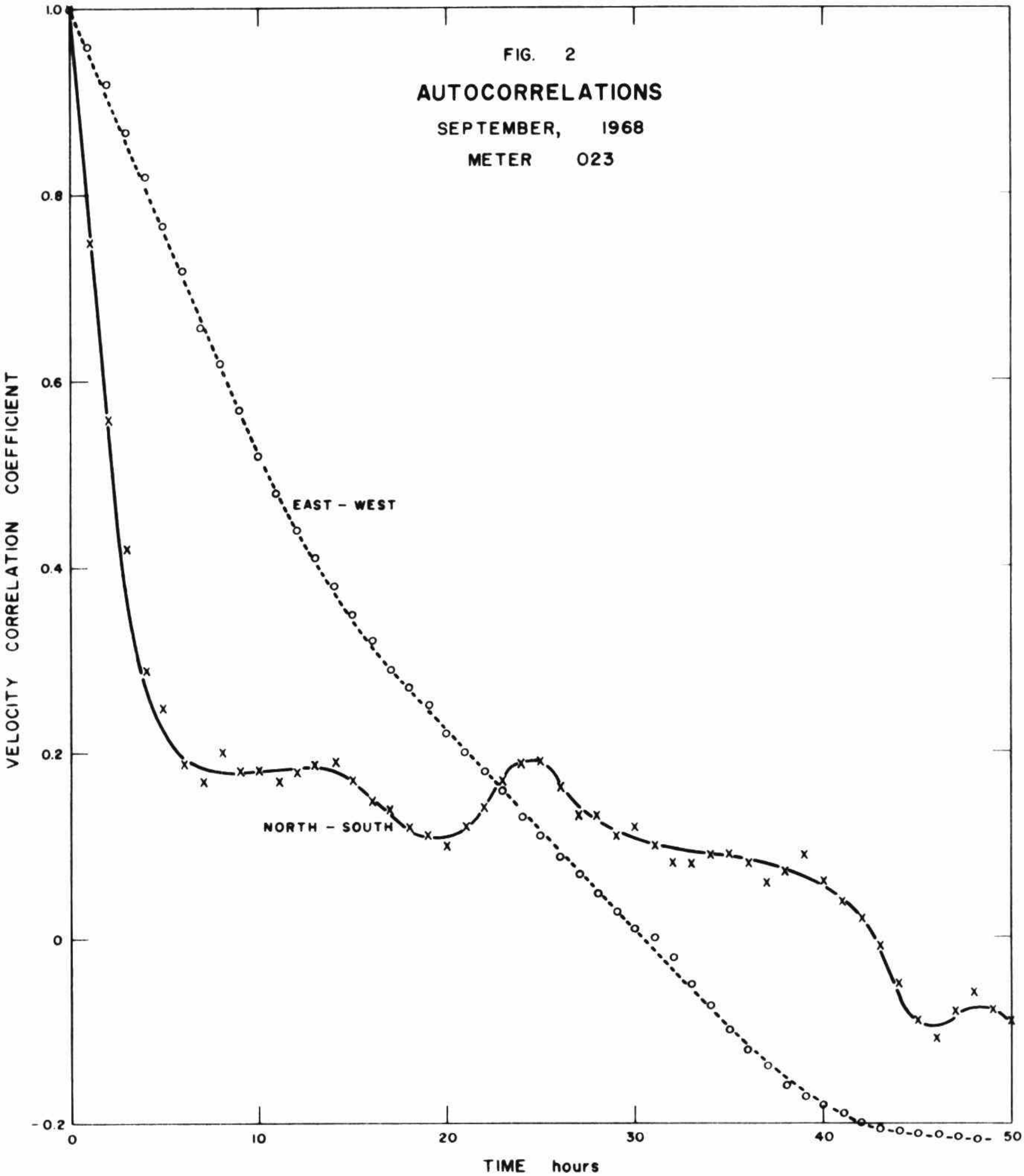
FIG. 2
AUTOCORRELATIONS
SEPTEMBER, 1968
METER 023
EAST - WEST
NORTH - SOUTH
VELOCITY CORRELATION COEFFICIENT
TIME hours
1.0
0.8
0.6
0.4
0.2
0
-0.2
0
10
20
30
40
50

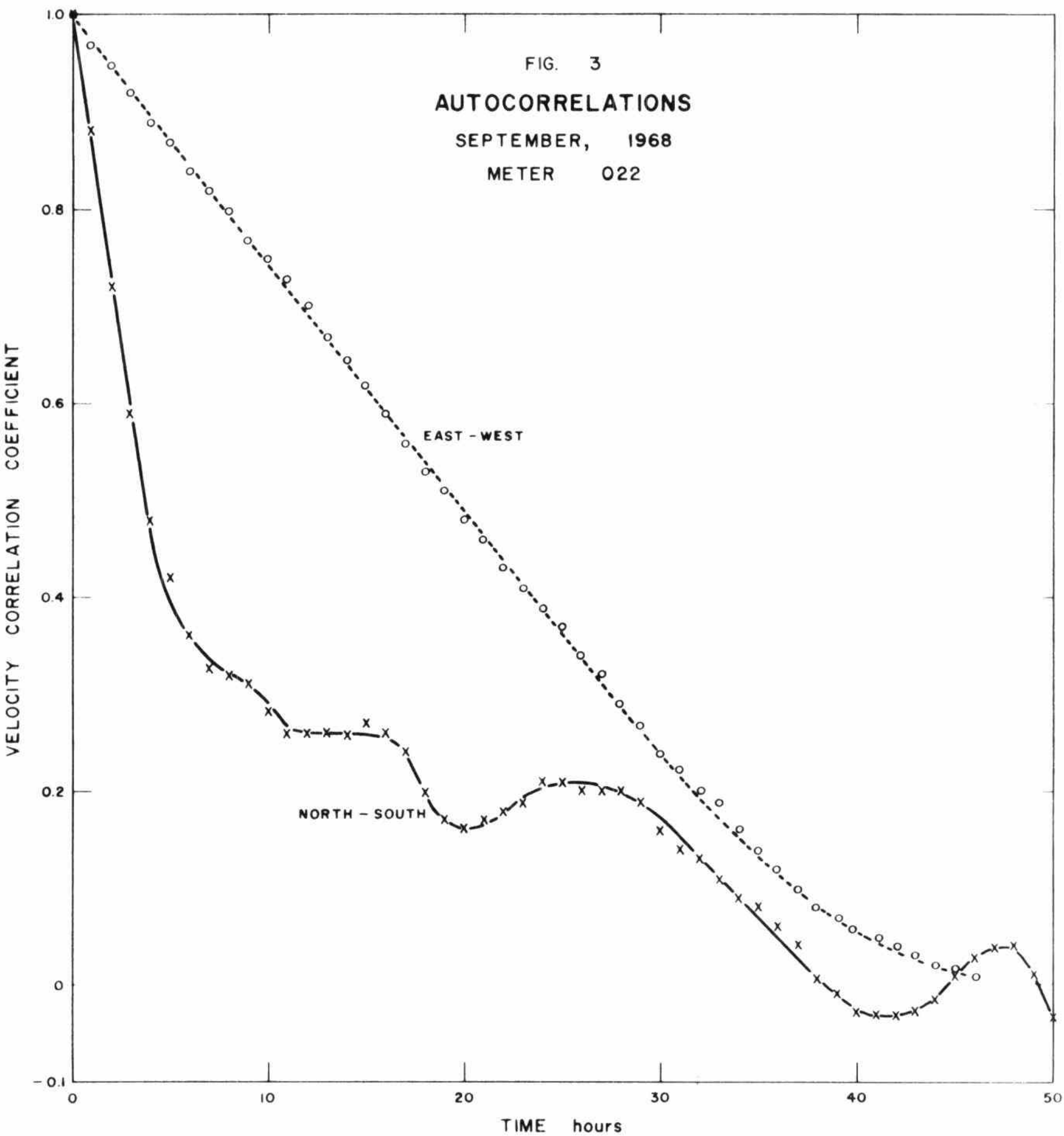

FIG. 3
AUTOCORRELATIONS
SEPTEMBER, 1968
METER 022
EAST - WEST
NORTH - SOUTH
VELOCITY CORRELATION COEFFICIENT
TIME hours
1.0
0.8
0.6
0.4
0.2
0
-0.1
0
10
20
30
40
50

distance away then comparing the results of the meters and by conducting dye dispersion experiments in the area of the meter.

Long Term Characteristics

By examining the history of the currents and comparing currents which occurred at different times, it is possible to obtain a measure of how the currents vary. This measure is called the auto-correlation function. If the current is steady and constant the auto-correlation function will be a simple constant equal to 1.0 otherwise it is a continuous function varying with the time interval considered. Examples of auto-correlation functions appear in Figures 2 and 3. These functions were obtained by searching the monthly records and comparing currents separated by different time intervals from zero to 50 hours (Palmer (11)). The dispersion pattern in an area is related to the auto-correlation function determined above as it is a functional relationship representing the currents in that area for a month. By integrating the auto-correlation functions from zero to 50 hours, it is possible to develop dispersion characteristics in two-dimensions for the month (see Appendix 1). The time limitation zero to 50 hours is dictated by the length of record available in a month (approximately 720 hours). However, it is representative as it includes most of the water movement activity variation and includes negative portions of the auto-correlation functions when reverse flow would tend to increase

concentration instead of diluting. The integrals of the autocorrelation functions are combined with the monthly resultant currents (details Appendix 1) to generate two-dimensional monthly average dispersion plumes (Figures 6-9). The auto-correlation integral represents the monthly current variation while the resultant monthly current represents the net effects of all the currents for a month. The concentration contours are plotted for a particular waste discharge of 5.42×10^5 cm^3/sec. The effect of using other flows can also be obtained. However, the transport capabilities of the measured currents in the area are limited if too large discharges of waste are introduced, the zone (dotted area surrounding meter positions) where there is no solution becomes larger because the measured currents are no longer applicable. The area will accept the larger discharges but the local lake currents will increase to accommodate the larger waste input. The effect of larger waste flows on an area can be measured in physical water movement terms by the size of the zone where no solution to the dispersion pattern is possible. If the zone of no solution is defined with a radius of 400 meters, the waste flows that can be accommodated by the existing currents are presented in Table 1.

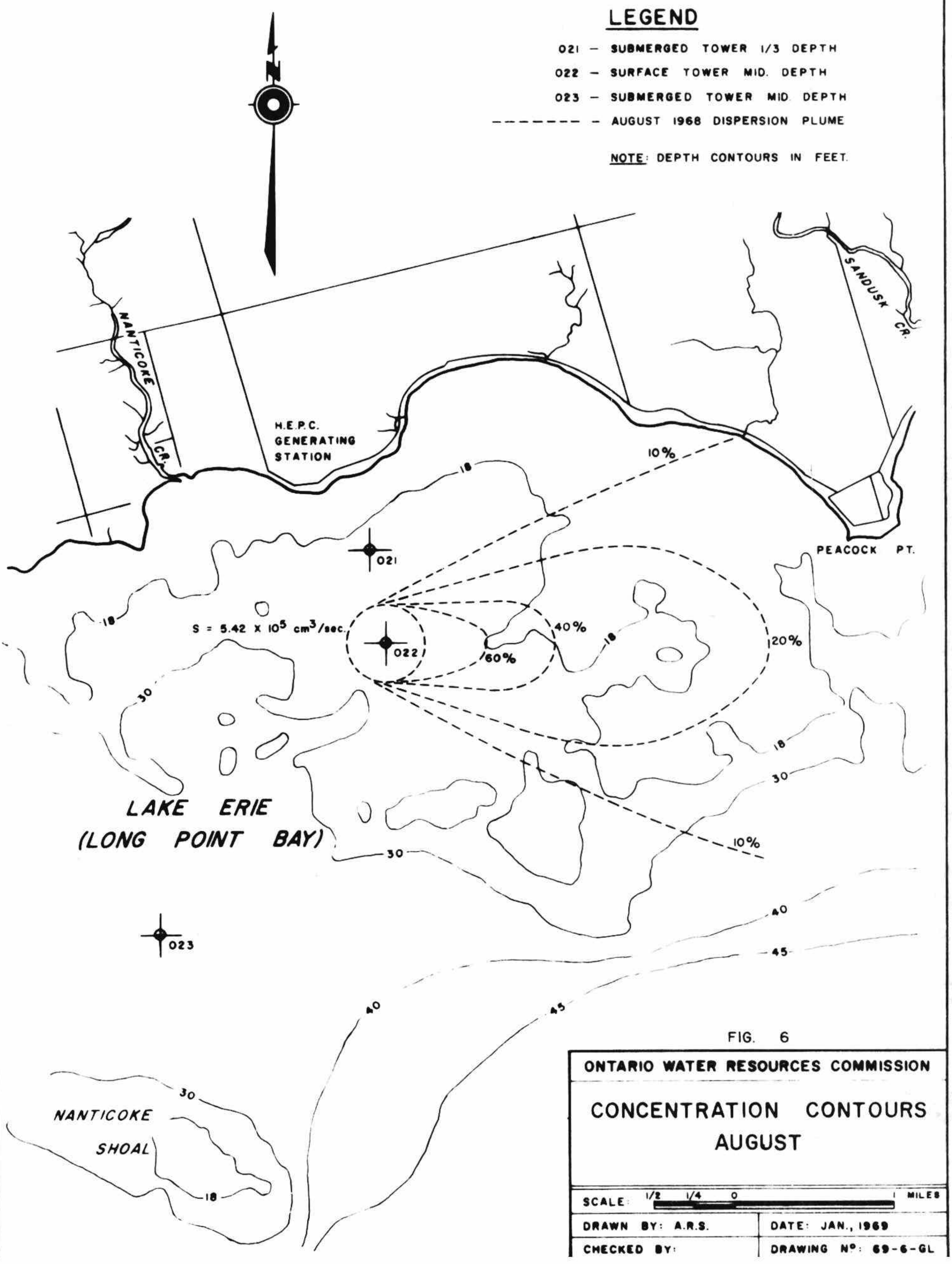

FIG. 6

ONTARIO WATER RESOURCES COMMISSION

CONCENTRATION CONTOURS
AUGUST

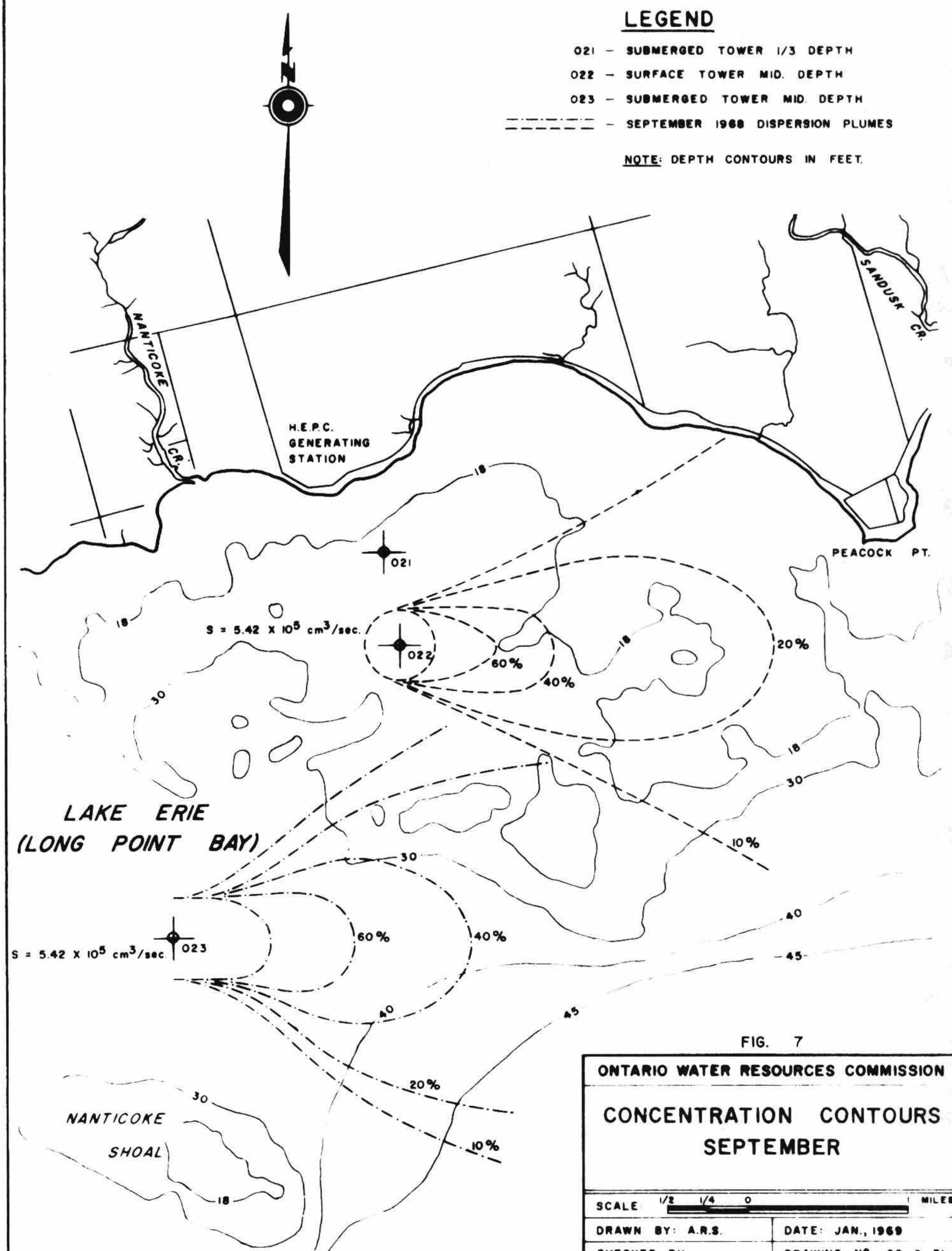

FIG. 7

ONTARIO WATER RESOURCES COMMISSION

CONCENTRATION CONTOURS
SEPTEMBER

SCALE: 1/2 1/4 0 1 MILES	
DRAWN BY: A.R.S.	DATE: JAN., 1969
CHECKED BY:	DRAWING Nº: 69-6-GL

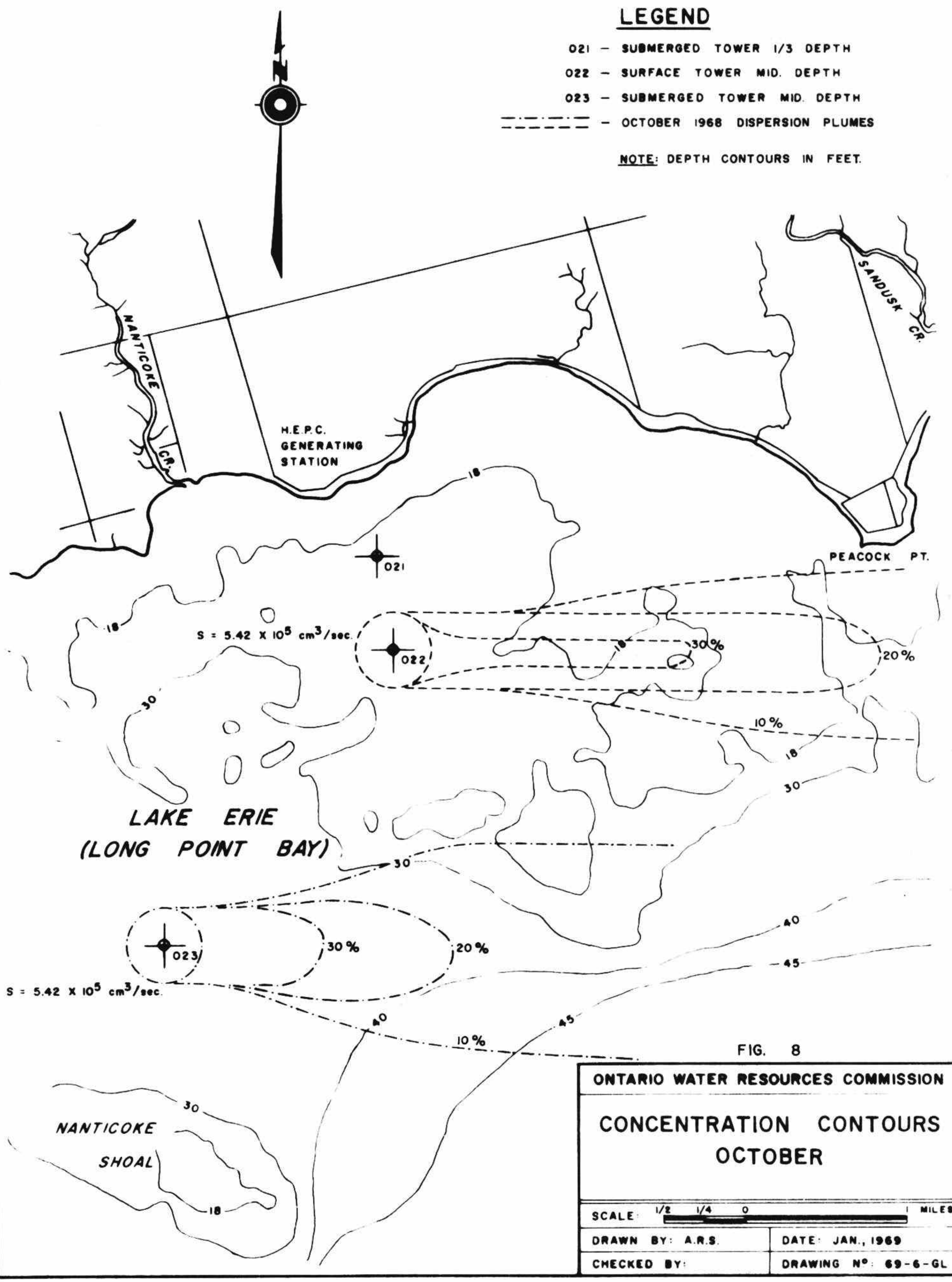
LEGEND
021 – SUBMERGED TOWER 1/3 DEPTH
022 – SURFACE TOWER MID. DEPTH
023 – SUBMERGED TOWER MID. DEPTH
– OCTOBER 1968 DISPERSION PLUMES
NOTE: DEPTH CONTOURS IN FEET.
NANTICOKE CR.
H.E.P.C. GENERATING STATION
SANDUSK CR.
PEACOCK PT.
021
022
023
S = 5.42 X 10⁵ cm³/sec.
S = 5.42 X 10⁵ cm³/sec.
30%
20%
10%
30%
20%
10%
18
30
40
45
LAKE ERIE (LONG POINT BAY)
NANTICOKE SHOAL
ONTARIO WATER RESOURCES COMMISSION
CONCENTRATION CONTOURS OCTOBER
SCALE: 1/2 1/4 0 1 MILES
DRAWN BY: A.R.S.
DATE: JAN., 1969
CHECKED BY:
DRAWING Nº: 69-6-GL

FIG. 8

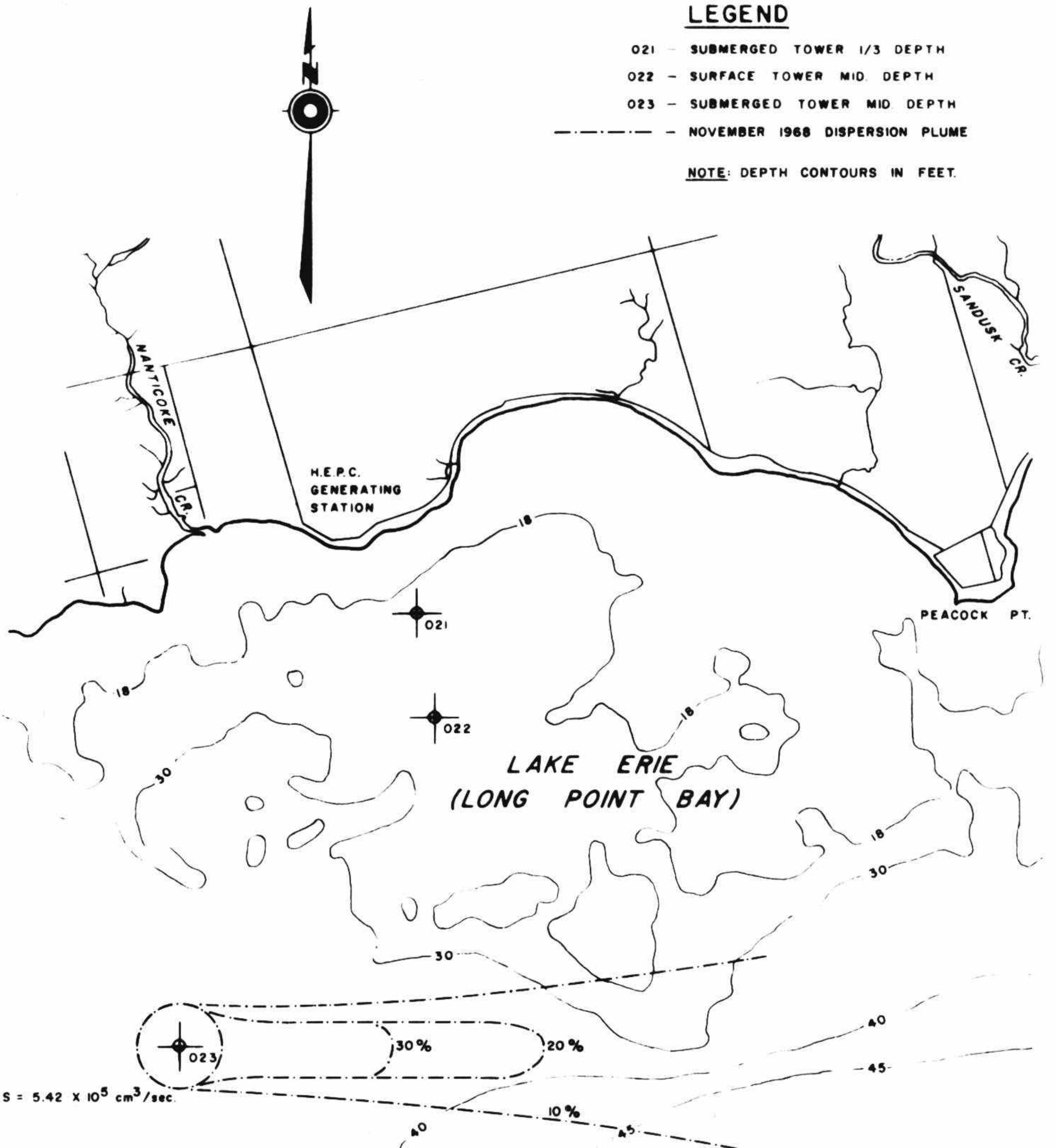

FIG. 9

ONTARIO WATER RESOURCES COMMISSION	
CONCENTRATION CONTOURS NOVEMBER	
SCALE 1/2 1/4 0 1 MILES	
DRAWN BY: A.R.S.	DATE: JAN., 1969
CHECKED BY:	DRAWING Nº: 69-6-GL

TABLE 1

WASTE INPUTS ALTERING CURRENT PATTERNS
UP TO 400 METERS FROM SOURCE

Month	Meter	Flow $cm^3/sec.$
August	022	5.53×10^5
September	022	5.53×10^5
	023	2.46×10^5
October	022	5.42×10^5
	023	7.76×10^5
November	023	5.45×10^5

Table 1 shows that the greatest and least assimilative or dispersion capabilities occur at Meter 023 in October and September respectively while conditions at Meter 022 remain reasonably constant for the three months. From a waste discharge and water intake point of view meter position 022 represents a better location on a long term basis as the conditions are more consistent.

The concentration contours plotted in Figures 6-9 have been extended to approximately 4,000 meters on the basis of cross-correlations between two meters 022 and 023. It was not considered practical to check the results with dye injection as it would be necessary to inject dye for a month and measure concentrations at a point frequently 24-hours a day. However, it will be possible to check the results with recording water quality meters.

This method assumes the dispersion pattern to be two-dimensional as all the measured data is two-dimensional. This is not a serious limitation provided the method is applied to the shallow near shore areas. However, it is not applicable in the presence of thermal stratification. The method further requires a continuous record of currents which is not easy to achieve in the field. Substituting missing values can be dangerous as the variation of the currents is large.

Short Term Dispersion Characteristics

The approach for the short term patterns must necessarily be different. It must consider all directions as dispersion occurs in all directions when any period of a few hours is considered. The relationships between currents in successive hours must be maintained. In other words, it is necessary to maintain the identity of hourly current values and their time of occurrence relative to other currents. One method that meets these requirements is a transitional state (current) probability matrix technique. If the state (current) is known now it is possible to predict the probability that it will be any other state in the next time period. As an example, suppose a die was thrown 1,000 times and a record was kept of the results. There are six possible outcomes (states) on each throw namely one, two, three --- or six. Suppose a two is cast, it is possible to predict the probability of all the other possible states occurring on the next throw by examining the record

to see what state followed a two state and how often it occurred. A different set of probabilities would be obtained by considering a different initial state - say one, three, etc. Considering all possibilities produces a transitional state probability matrix six by six, which is sometimes called a first order Markov chain process as it considers values in successive time intervals. Higher order matrices can be determined by considering states two time intervals away, then three time intervals, etc. These matrices are higher order chains. The short term method establishes 80 current states consisting of 8 different directions and 10 different speeds (Appendix 2). The first order Markov chain or transition state probability matrix is determined (Kemeny (13)) for a month's record of currents. If the initial current state is known, it is now possible to predict the probability of all other states occurring in the next hour.

The short term dispersion patterns required are for approximately six hours. Short term dye dispersion runs normally extend over a period of approximately six hours. For the nearshore areas a six hour period would include all seiche (wind generated water movement which pulsates similar to a wave in a bathtub) effects generated by local shoreline configuration and diurnal effects which are prevalent in the summer months. To simulate this six hour pattern, it would be necessary to develop five higher order probability matrices and combine them

hour by hour. This is beyond the limits of storage on the computer consequently, the first order matrix was applied five times for each successive hour state. All five-step sequences were tabulated and the final five-step probabilities were obtained by a fifth order state transitional matrix (Bharucha-Reid (14)). From the sequence of steps (resolved in the four major compass directions), it is possible to obtain the distance travelled in five hours. The weighted mean distances travelled can then be determined by using the five hour distances travelled and the transition probabilities as a weight factor. Similar weighted standard deviations in each direction can be obtained. These standard deviations can be utilized to represent the mean dispersion characteristics in each direction (see Appendix 2), the results show the probability of dispersion occurring in the four major compass directions for five successive hours, the maximum, mean and minimum distances travelled and the dispersion characteristics by direction for certain waste discharges (see Figures 10 to 13). As with the long term method, if the waste input flow is too large, it will change the local currents from those measured restricting the application of the prediction equations. Consequently, the method can be used to indicate the loadings required to change the local water movement characteristics.

The results produced by this method are useful for positioning intakes and outlets such as to minimize interference between the two.

FIG. 10

METER 022 - SEPTEMBER, 1968

FIVE - HOUR DILUTION CHARACTERISTICS OF A CONTINUOUS DISCHARGE OF 78,000 cm^3 / sec.

NORTH

PROB.* = 0.040
DISTANCE - 5-HOUR
MAX. = 534 meters
MEAN = 385 m
MIN. = 254 m
DILUTION MEAN 5-HOUR
CONC. = 0.38 ORIG. CONC.

WEST

PROB. = 0.329
DISTANCE - 5-HOUR
MAX. = 2,484 meters
MEAN = 880 m
MIN. = 76 m
DILUTION MEAN 5-HOUR
CONC. = 0.072 ORIG. CONC.

EAST

PROB. = 0.499
DISTANCE - 5-HOUR
MAX. = 2,700 meters
MEAN = 1,010 m
MIN. = 51 m
DILUTION MEAN 5-HOUR
CONC. = 0.055 ORIG. CONC.

SOUTH

PROB. = 0.130
DISTANCE - 5-HOUR
MAX. = 942 meters
MEAN = 237 m
MIN. = 0 m
DILUTION MEAN 5-HOUR
CONC. = 1.0 ORIG. CONC.

* PROB. = PROBABILITY THAT THE WATER MOVEMENT WILL BE IN THIS DIRECTION FOR 5 SUCCESSIVE HOURS.

FIG. 11

METER 022 - OCTOBER, 1968

FIVE - HOUR DILUTION CHARACTERISTICS OF A CONTINUOUS DISCHARGE OF 100,000 cm^3 / sec.

NORTH

PROB.* = 0.062
DISTANCE - 5-HOUR
MAX. = 2,482 meters
MEAN = 547 m
MIN. = 16 m
DILUTION MEAN 5-HOUR
CONC. = 0.21 ORIG. CONC.

WEST

PROB = 0.235
DISTANCE - 5-HOUR
MAX. = 3,510 meters
MEAN = 1,250 m
MIN. = 0 m
DILUTION MEAN 5-HOUR
CONC. = 0.059 ORIG. CONC.

EAST

PROB. = 0.637
DISTANCE - 5-HOUR
MAX. = 3,510 meters
MEAN = 1,440 m
MIN. = 153 m
DILUTION MEAN 5-HOUR
CONC. = 0.051 ORIG. CONC.

SOUTH

PROB. = 0.057
DISTANCE - 5-HOUR
MAX. = 2,482 meters
MEAN = 177 m
MIN. = 0 m
DILUTION MEAN 5-HOUR
CONC. = 1.0 ORIG. CONC.

* PROB. = PROBABILITY THAT THE WATER MOVEMENT WILL BE IN THIS DIRECTION FOR 5 SUCCESSIVE HOURS.

FIG. 12

METER 023 - SEPTEMBER, 1968

FIVE - HOUR DILUTION CHARACTERISTICS OF A CONTINUOUS DISCHARGE OF 38,600 cm^3 / sec.

NORTH

PROB.* = 0.03
DISTANCE - 5-HOUR
MAX. = 331 meters
MEAN = 167 m
MIN. = 25 m
DILUTION MEAN 5-HOUR
CONC. = 1.0 ORIG. CONC.

WEST

PROB. = 0.23
DISTANCE - 5-HOUR
MAX. = 2,340 meters
MEAN = 544 m
MIN. = 25 m
DILUTION MEAN 5-HOUR
CONC. = 0.065 ORIG. CONC.

EAST

PROB. = 0.48
DISTANCE - 5-HOUR
MAX. = 2,484 meters
MEAN = 957 m
MIN. = 0 m
DILUTION MEAN 5-HOUR
CONC. = 0.035 ORIG. CONC.

SOUTH

PROB. = 0.26
DISTANCE - 5-HOUR
MAX. = 2,340 meters
MEAN = 270 m
MIN. = 0 m
DILUTION MEAN 5-HOUR
CONC. = 0.378 ORIG. CONC.

* PROB. = PROBABILITY THAT THE WATER MOVEMENT WILL BE IN THIS DIRECTION FOR 5 SUCCESSIVE HOURS.

FIG. 13

METER 023 - OCTOBER, 1968

FIVE - HOUR DILUTION CHARACTERISTICS OF A CONTINUOUS DISCHARGE OF 348,000 cm^3 / sec.

NORTH

PROB.* = 0.105
DISTANCE - 5-HOUR
MAX. = 2,558 meters
MEAN = 767 m
MIN. = 38 m
DILUTION MEAN 5-HOUR
CONC. = 0.43 ORIG. CONC.

WEST

PROB. = 0.225
DISTANCE - 5-HOUR
MAX. = 2,558 meters
MEAN = 1,570 m
MIN. = 38 m
DILUTION MEAN 5-HOUR
CONC. = 0.27 ORIG. CONC.

EAST

PROB. = 0.557
DISTANCE - 5-HOUR
MAX. = 3,942 meters
MEAN = 1,000 m
MIN. = 0 m
DILUTION MEAN 5-HOUR
CONC. = 0.14 ORIG. CONC.

SOUTH

PROB. = 0.110
DISTANCE - 5-HOUR
MAX. = 2,329 meters
MEAN = 360 m
MIN. = 0 m
DILUTION MEAN 5-HOUR
CONC. = 1.0 ORIG. CONC.

* PROB. = PROBABILITY THAT THE WATER MOVEMENT WILL BE IN THIS DIRECTION FOR 5 SUCCESSIVE HOURS.

The results can also be used to predict the effect of a discharge on other local points on the shoreline. The mean dispersion characteristics will provide the mean concentration that will occur with the probability tabulated. However, individual maximum readings can be expected two to three times larger than the mean value (Gifford (15)). Generally, these high values will only occur for a very short period of time.

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APPENDIX 1

METHOD

FOR

LONG TERM DISPERSION

CHARACTERISTICS

APPENDIX 1

LONG TERM DISPERSION CHARACTERISTICS

Development of Method

J. O. Hinze (2), in his book entitled "Turbulence", presented various forms of dispersion equations for different types of flow fields. All equations contain terms representing turbulence characteristics, distances from sources and mean velocities where applicable. The problem is thus to select the appropriate equations for the Nanticoke area.

Measured velocities using current meters are two-dimensional, consequently, any relationship describing the dispersion characteristics will have to be two-dimensional. This is not a serious limitation since the maximum depth in the study area is only 13m and provided the long term distribution of a passive contaminant is considered. Over a long period of time the distribution with depth will be small compared to the horizontal (parallel to the water surface) distribution. Energy spectra for the north-south and east-west correlations between meter locations indicated that the dispersion characteristics for the components are different. Therefore, the applicable expression must differentiate between the components. One equation that meets these requirements is:

$$P\left(x_1, x_2\right) = \int_0^{\infty} dt \, \frac{S}{(2\pi)^{3/2}} \, \frac{}{\left(\overline{y_1^2} \; \overline{y_2^2}\right)^{1/2}} \exp\left[-\tfrac{1}{2}\left(\frac{\left[x_1 - \overline{U}_1 \, t\right]^2}{\overline{y_1^2}} + \frac{x_2^2}{\overline{y_2^2}}\right)\right] \quad \text{(1)}$$

(Hinze (2), p. 327)

where:

x_1	=	distance from source in east-west direction in cm.
x_2	=	distance from source in north-south direction in cm.
S	=	continuous point source cm^3/sec.
$\overline{y_1^2}$	=	variance of the displacement in the east-west direction in cm^2.
$\overline{y_2^2}$	=	variance of the displacement in the north-south direction in cm^2.
$\overline{U}_1$	=	mean velocity in the east-west direction in cm/sec.
$P(x_1, x_2)$	=	probability of finding a marked fluid particle at a point x_1, x_2.
t	=	time in secs.

Equation (1) is a special solution of the general equation (Hinze (2), p. 312):

$$\frac{\partial}{\partial t} P\left(x_1, x_2, x_3, t\right) = \epsilon_{ij} \frac{\partial}{\partial x_i \partial x_j} P\left(x_1, x_2, x_3, t\right) \quad \text{(2)}$$

where ϵ_{ij} is the diffusion coefficient tensor of second order. This is the classical diffusion equation for molecules and heat in four-dimensional space which states that quantities move down concentration gradients.

Now $\epsilon_{ij} = \frac{1}{2} \frac{d}{dt} \overline{y_i y_j}$ so that the diffusion coefficient is a function of the space and time. To obtain dispersion plumes from the two-dimensional (north-south and east-west) current meter records, it is necessary to assume that the dispersion is described by the two horizontal diffusion coefficients, and that the coefficients remain constant throughout the plume. The first assumption of two-dimensionality is not serious, provided that reasonably long time periods are considered so that the distribution with depth is uniform, or conversely that the distances are far enough from the source such that the depth scale of 13m is not significant. The second assumption that the diffusion coefficients are constant with distance from the source can be justified by developing the coefficients as long time averages and invoking the condition that the velocity field is reasonably uniform over areas of two or three miles. The uniform velocity field assumption has been verified at Nanticoke by the time series analysis of velocities which revealed significant correlations between meters 022 and 023 (Palmer (7)). It was decided to restrict the assumption of constant diffusion coefficient to distances of 5×10^5 cm or 50 integral scales (Lumley (3)).

For short times (Hinze (2), p. 332)

where $\tau < \zeta_L$

τ = time
ζ_L = Lagrangian time scale

$$\overline{y_2^2} = \frac{\overline{u_2^2}\ x_1^2}{\overline{U_1^2}} \quad \text{(3)}$$

For long times (Ibid)

where $\frac{\epsilon}{\overline{U}_1 x_1} << 1.$

$$\overline{y_2^2} = \frac{2\ \epsilon\ x_1}{\overline{U}_1} \quad \text{(4)}$$

and $\epsilon = u' \Lambda_L$ (5)

u' = fluctuating velocity root mean square in cm/sec.

Λ_L = Lagrangian integral scale in cm.

Since the meters are fixed, the velocity data is Eulerian not Lagrangian. However, for extremely large Reynold's numbers, it can be shown by dimensional reasoning (Lumley (3), p. 34) that:

$$\Lambda_L \equiv \overline{U}\ \zeta_E$$

where ζ_E = Eulerian integral time scale

The Reynold's number for Nanticoke is:

$$Re = \frac{\overline{U}D}{\nu} = 2.05 \times 10^6$$

where $\overline{U}$ = mean velocity in cm/sec.

D = depth in cm.

ν = kinematic viscosity cm^2/sec.

By definition

$$\zeta_E = \int_0^{\infty} dt \ \frac{\overline{u(t')u(t'-t)}}{u'^2} = \int_0^{\infty} dt \ R(t) \quad \text{.......................................} (6)$$

where $R(t)$ = correlation coefficients

ζ_E = integral of the correlation coefficients with respect to time in secs.

u = fluctuating velocity in cm/sec.

The current meter data was analyzed on an hourly basis to minimize alaising in the energy spectra. Consequently, correlation coefficients were developed for hourly time lags. The question arises as to whether hourly lags are appropriate for determining $R(t)$?

Hinze (7), p.4) indicates a sampling time interval:

$$T = \frac{\text{length scale}}{\text{mean velocity}}$$

Using the depth of 13m:

$$T = 7 \text{ minutes}$$

On the other hand, Lumley (3), p.39) indicates a sample time interval of:

$$T \approx 200 \times \text{integral scale}$$

Okubo (5) estimated the integral scale to be approximately 2m for Lake Erie:

$$T = \frac{200 \times 200}{3 \times 60} = 220 \text{ minutes}$$

Since Hinze's intervals are basically laboratory oriented while Lumley's are meteorologically oriented the differences are likely a result of the scale of the processes. The Nanticoke characteristics would obviously tend towards the meteorological scale. Consequently, it seems reasonable to utilize hourly lag intervals for the correlation coefficient.

To evaluate the integral in equation (6), the limits must be reduced from infinity due to noise and very large scale motions. An upper limit of 50 hours was selected (see Figs. 2 and 3) as covering the time interval of major interest. As it is necessary to consider long time periods for the two-dimensional assumption, it was decided to use a month as a convenient time as the basis for determining dispersion patterns.

Velocities in the Nanticoke area were small (monthly averages for Meters 023 and 022 were 1.7 and 4.7 cm/sec. respectively) with complete reversals of direction frequently occurring within 2 or 3 days (see Figs. 3a and b, Palmer (7)). This is typical of near shore areas on lakes Erie and Ontario. Consequently, it would be extremely difficult to consider dispersion for short time periods. A fundamental time period of a month permits the averaging of many reversals. Obviously, if the currents during a month were very persistent in one direction (with a persistence factor of 0.8 or greater, it would be

possible to consider shorter time periods with accuracy.

The monthly resultant current vector was used for the mean velocity $\overline{U}_1$ and the standard deviations of the monthly currents were used for estimates of the fluctuating velocities u'. The corresponding persistence factors provide an indication of the probability of the dispersion pattern occurring in some other direction during any day of the month. The values thus derived are tabulated below:

TABLE 1

SUMMARY OF CHARACTERISTIC DIFFUSION QUANTITIES
FOR SEPTEMBER, 1968

Meter	ζ_E North hours	ζ_E East hours	Λ_L North cm	Λ_L East cm	u' North cm/sec.	u' East cm/sec.
022*	9.6	19.5	8.6×10^3	1.46×10^4	1.49	4.49
023*	7.25	9.5	3.62×10^3	4.02×10^4	0.65	3.08

	U North cm/sec.	U East cm/sec.	Persistence Factor	ϵ North cm²/sec.	ϵ East cm²/sec.
022	0.25	2.08	0.44	1.28×10^4	7.19×10^5
023	0.14	1.19	0.70	2.36×10^3	1.25×10^5

* for meter locations see Fig. 1.

TABLE 2

RESULTS OF OTHER STUDIES

Reference	ϵ (cm^2/sec.)
Csanady (1)	4×10^2 (one at 2×10^3)
Okubo (5)	3 to 6×10^4
Noble (4)	2.44×10^2
Palmer (6)	0.65 to 1.3×10^2

The above results compare favourably with Okubo (5) who used drogues at similar distances offshore but are significantly larger than the other studies. However, as the other studies are either surface dye dispersion determinations or evaluations in deeper water, it is felt that a comparison is not valid. A very important feature of the Nanticoke results is that they portray the nearshore processes more precisely with two-dimensional dispersion coefficients.

The dispersion in September is more intense nearer the shore (Meter 022) and parallel (east) to the shore. Both the fluctuating velocities and the integral scales by the preceding method are much larger than those determined by Okubo (5) who found the fluctuating velocities to be 0.3 cm/sec. and the integral scale one to 10 metres. It is certainly unlikely that the integral scale would be as small as Okubo estimates. Significant correlations exist between velocities at Meters 022 and 023 (Palmer (5)). The discrepancy in the fluc-

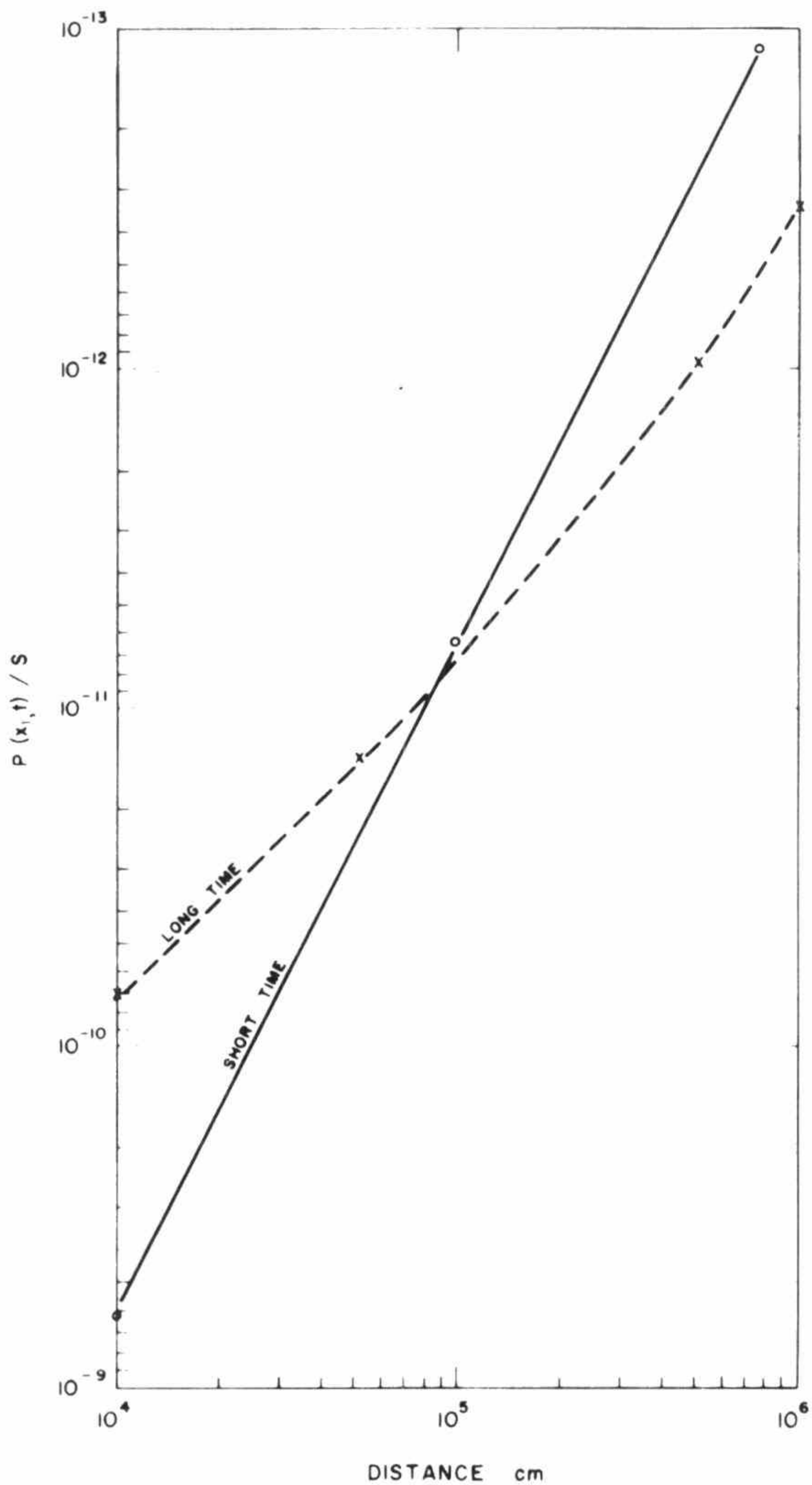

FIG. 4

COMPARISON OF LONG AND SHORT TIME DIFFUSION PROBABILITIES

tuating velocities is due to the different time periods considered.

The main area of concern for the dispersion plume is from 4×10^4 cm to 5×10^5 cm (1/4 miles to 3 miles) from the source (waste discharge point). Considering that the short time formula (equation (3)) is limited to distances up to approximately 10^2 cm while the long time formula (equation (4)) is limited to distances beyond 10^7 cm, the area of interest is somewhere between these limits.

The short and long time forms of equation (1) (Hinze (2), p.327 and p.329) for $x_2 = 0$ are plotted in Figure 4. It can be seen that the area of interest is centred about the intersection of the short and long time forms of the equation. Thus, it is reasonable to use an average of the short and long time equations for this area. Short and long time equations for $\overline{y^2}$ in two-dimensions (equations (3) and (4)) were computed for each computation point on a square grid (see Fig. 1) and averaged. The values thus determined were then substituted into equation (1) which was then integrated numerically on the computer until the last step added less than half a per cent of the total. (The integral equation (1) converges rapidly with time intervals of 15 hours).

<u>Sample Calculation</u>

The numerical solution of equation (1) generates values of P/S for various points on a two-dimensional grid (Fig. 5). For a P not greater

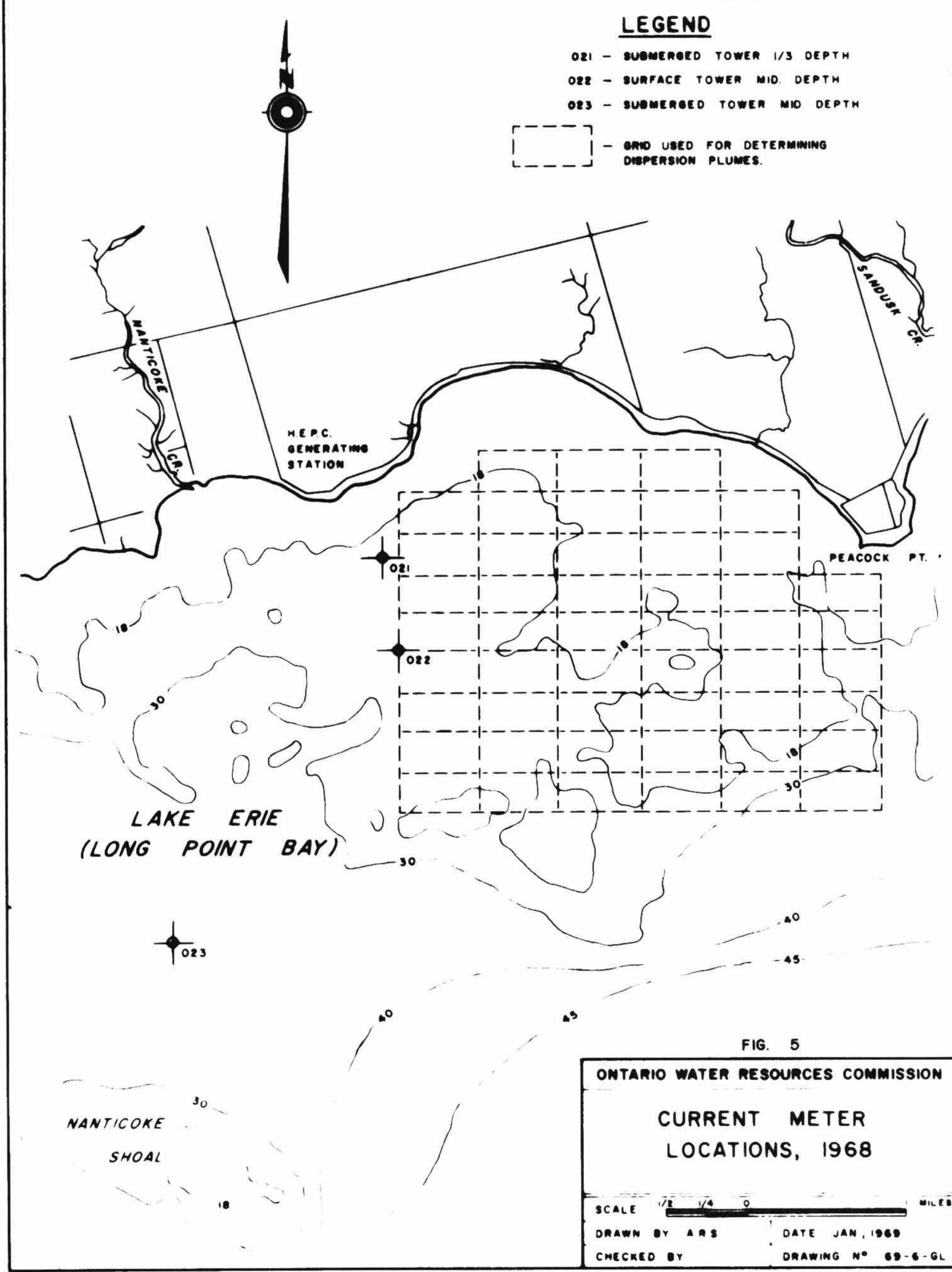

FIG. 5

ONTARIO WATER RESOURCES COMMISSION

CURRENT METER LOCATIONS, 1968

SCALE 1/2 1/4 0 MILES

DRAWN BY A R S | DATE JAN, 1969

CHECKED BY | DRAWING Nº 69-6-GL

than 1.0 at $x_1 = 4 \times 10^4$ cm and $x_2 = 0$, the maximum point source strength is 5.53×10^5 $cm^3/sec.$ (approximately 24 cfs) for Meter 022 and "S" of 2.46×10^5 $cm^3/sec.$ (approximately 10 cfs) for Meter 023. This means that for distances less than 4×10^4 cm from the source it must be assumed that a uniform concentration equivalent to source strength exists since the equations do not apply in this range. For larger flows, the area of uniform concentration would be greater. This is not unexpected since the equations reflect the fact that larger sources will change the local mean velocities $\overline{U}_1$ in the immediate area to accommodate the larger flows. The mean velocities determined by current meter studies can therefore not be used when no actual source is operative in this area. The September dispersion plumes for a source strength $S = 5.42 \times 10^5$ $cm^3/sec.$ and $S = 5.42 \times 10^5$ $cm^3/sec.$ at Meters 022 and 023 respectively are plotted in Figure 7 as percentages of the source strength. This means that if the source strength is 10 ppm, the concentration on the 50 per cent contour would be 5 ppm.

Results

The results for the meters in August, October and November are tabulated below. The diffusion coefficients for November agree with the results obtained by a conventional drogue study carried out at the same time farther offshore (Hamblin (8)). Meter 023 was not installed until September and there were only 10 days of good data for Meter 022

in November. Consequently, the record is too short to produce reliable results and these values have been omitted.

TABLE 3

SUMMARY OF CHARACTERISTIC DIFFUSION QUANTITIES

Month	Meter	ζ_E North hours	ζ_E East hours	Λ_L North cm	Λ_L East cm	u' North cm/sec.	u' East cm/sec.	U North cm/sec.	U East cm/sec.	Persistence Factor	ϵ North $cm^2/sec.$	ϵ East $cm^2/sec.$
August	022	5.98	3.86	6.75×10^3	31.2×10^3	1.90	4.45	0.312	2.24	0.54	1.28×10^4	1.39×10^5
September	022	9.60	19.5	8.6×10^3	1.46×10^4	1.49	4.94	0.25	2.08	0.44	1.28×10^4	7.19×10^5
	023	7.25	9.5	3.62×10^3	4.02×10^4		3.08	0.14	1.19	0.70	2.36×10^3	1.25×10^5
October	022	4.21	9.86	0.99×10^3	1.78×10^5	1.20	6.59	0.065	4.99	0.75	1.20×10^3	1.17×10^6
	023	17.52	25.52	7.85×10^3	3.33×10^5		7.86	0.124	3.62	0.54	1.46×10^4	2.62×10^6
November	023	13.67	28.1	1.27×10^3	4.82×10^5		10.90	0.257	4.76	0.37	1.95×10^3	5.26×10^6

The corresponding dispersion plumes are plotted in Figures 6, 7, 8 and 9 using the source strengths (S) of 5.42×10^5 cm^3/sec. Table 4 lists the size of the sources that the existing currents can accommodate without changing the currents beyond 4×10^4 cm from the source.

TABLE 4

SOURCE STRENGTHS USED IN DISPERSION PLUME COMPUTATIONS

Month	Meter	S cm^3/sec.
August	022	5.53×10^5
September	022	5.53×10^5
	023	2.46×10^5
October	022	5.42×10^5
	023	7.76×10^5
November	023	5.45×10^5

Conclusions

The preceding method for predicting the long term dispersion plumes provides a comprehensive portrayal of the nearshore processes. It permits differentation of the dispersion characteristics in two-dimensions for various locations offshore. By using the continuous records from current meters, the method considers the time dependent

and periodic nature of the currents serially. The periodic nature of the current regime necessitates the development of average conditions over reasonable time periods. Consequently, the resulting plumes (see Figs. 6, 7, 8 and 9) do not represent the dispersion at any particular time, but are the average of conditions occurring throughout the month. The method also predicts how far a new source will alter the current patterns in the area.

The assumptions of the method dictate the extent of its applications. The expression is restricted to areas where the Reynold's numbers are large and where the assumption of constant long term effective diffusion coefficient is valid.

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NOTATION

D	=	depth;
$P(x_1, x_2)$	=	probability of finding a marked fluid particle at x_1, x_2;
$R(t)$	=	correlation coefficient;
Re	=	Reynold's number;
S	=	continuous point source;
t	=	time;
T	=	sampling time interval;
$\overline{U_1}$	=	mean velocity (east-west);
u'	=	root mean square fluctuating velocity;
u	=	fluctuating velocity;
x_1, x_2, x_3	=	co-ordinate directions;
$\overline{y_1^2}$	=	variance of the displacement in east-west direction;
$\overline{y_2^2}$	=	variance of the displacement in north-south direction;
ϵ_{ij}	=	diffusion coefficient tensor of second order;
ϵ	=	one-dimensional diffusion coefficient;
π	=	3.141593;
Λ_L	=	Lagrangian integral space scale;
ζ_L	=	Lagrangian integral time scale;

ζ = Eulerian integral time scale;

τ = time; and

ν = kinematic viscosity

All directions are coming from for currents.

APPENDIX 2

METHOD FOR SHORT TERM

DISPERSION CHARACTERISTICS

APPENDIX 2

SHORT TERM DISPERSION CHARACTERISTICS

Development of Method

Markov chain transition probability matrices for successive sequential hourly intervals were obtained by analyzing current meter records for a month, in terms of the magnitude and direction of the current. Ten magnitude classes of width 2 or 3 cm/sec. depending on the velocity range starting with 0 cm/sec. were selected. This produced a 10 x 10 probability transition matrix designated M_{ij} (see Table 1) where i represents the initial current magnitude class and j represents the current magnitude class at a time period of one hour later. Similarly, eight current direction classes of width 45° starting at 337.5° were selected and designated D_{KL} (see Table 2). The mid-points of the angle classes represent the eight primary compass directions. Thus, each element in a row represents one possible final state for the initial state specified by that row. The probabilities were found by searching all sequential two hour current meter readings pairs for a month. The total number of transitions from any particular state in the first hour were recorded and then used to divide the number which reached any particular state an hour later to give the probability. Combining the magnitude and direction results in 80 possible states, eight directions for each of the ten possible magnitudes. For convenience

the states were numbered 1 to 80 as shown in Table 1.

TABLE 1

STATE DEFINITIONS

Direction

Magnitude	337.5° to 22.5°	22.5° to 67.5°		247.5° to 292.5°	292.5° to 337.5°
0 -2 cm/sec	1	2		7	8
2 - 4 cm/sec	9	10		15	16
-	-	-		-	-
-	-	-		-	-
-	-	-		-	-
16 - 18 cm/sec	65	66		71	72
18 - 20 cm/sec	73	74		79	80

If an initial state of a current is known (e.g. magnitude and direction), it is possible to predict the probability of any other state occurring in the next hour from the 80 x 80 matrix produced by the following operation:

$T_{mn} = M_{ij} \times D_{KL}$ .... (1)

Here "m" represents the initial state specified by the magnitude "i" and angle "k" and "n" represents the final state specified by "j" and "l". This may be seen more clearly from Table 2.

TABLE 2

ONE HOUR TRANSITION PROBABILITY MATRIX

Final State

	SPEED		0 to 2 cm/sec.			2 to 4 cm/sec.			-
		ANGLE	337.5^{o} to 22.5^{o}	- - -	292.5^{o} to 337.5^{o}	337.5^{o} to 22.5^{oo}	- - -	292.5^{o} to 337.5^{o}	- - -
Initial State	0 to 2 cm/sec.	337.5-22.5 - - - - - - 292.5-337.5					A		
	2 to 4 cm/sec.	337.5-22.5 - - - - - - 292.5-337.5							
							etc.		

Each block represents an 8 x 8 matrix and a sample representing block A may be found in Table 3.

TABLE 3

ONE HOUR TRANSITION PROBABILITY MATRIX

Initial State 0 to 2.0 cm/sec.		Final State 2.0 to 4.0 cm/sec.			
		337.5^{o} to 22.5^{o}	22.5^{o} to 67.5^{o}	-------	292.5^{o} to 337.5^{o}
	337.5^{o} to 22.5^{o}	0.01737	0.00668	-------	0.00240
	22.5^{o} to 67.5^{o}	0.00634	0.01692	-------	0.00121
	- - -	- - -	- - -	-------	- - -
	292.5^{o} to 337.5^{o}	0.00626	0.00521	-------	0.01147

Hence, by selecting an initial current a row is specified and the probability of any state one hour later may be found by looking at the intersection of that row and the appropriate column. For example, the first element of block "A" represents the probability of going from a state of 0-2 cm/sec. and 337.5 to 22.5 to a state of 2-4 cm/sec. and 337.5 to 22.5^{o}.

It was observed that for any given initial state the probability of the successive state dropped to zero within $\mp$ 24 states of the initial

state. This represents the continuity characteristics of the current pattern on an hourly basis. It is now possible to expand the first order Markov chain to sequences of state changes over five hour intervals hour by hour. This may be done by multiplying together the probabilities for each of the five steps. For instance: PR (state 1 to state 1 to state 1 to state 1 to state 2 to state 2) = PR (1 to 1) x PR (1 to 1) x PR (1 to 1) x PR (1 to 2) x PR (2 to 2). There are approximately $(80)^5$ such paths possible for a five hour sequence. Due to computer space and time limitations transitions were limited to $\pm$ 8 states from the initial state. It was necessary to limit the probability when considering the five step sequences for computer running time. The processing of one month's records for two meters with a five step probability greater than 0.001 required a core space of 250K bytes and had a running time of 1.50 hours on an IBM 360/65 installation. The sum of the transition probabilities must be equal to 80.0. The probability limit was changed to obtain different sums of the transitional probabilities to establish an acceptable probability limit. A sample of the results for various probability limits is presented in Table 4.

TABLE 4

PROBABILITY LIMIT COMPARISON
METER 023
OCTOBER 1968

Probability Limit	0.001	0.0001	0.000001
Sum of Transition Probabilities	2.0	21.9	29.5
$\sqrt{\bar{y}^2}$ cm.			
North	4.65×10^3	7.0×10^4	7.82×10^4
East	3.08×10^3	9.23×10^4	9.34×10^4
South	4.5×10^3	7.5×10^4	7.35×10^4
West	3.42×10^4	1.16×10^5	1.15×10^5

The results in Table 4 indicate that a probability limits of 0.0001 accounts for an appropriate number of sequences to be indicative of the mean spread. Consequently, a probability level of 0.0001 was selected as a criteria for determining the short term dispersion characteristics. For each sequence of states the distance travelled in the five hours can be resolved into north-south and east-west components. A sample of part of the output showing the probabilities, distance travelled and the sequence of states is presented in Table 5.

TABLE 5

FIVE HOUR SEQUENTIAL STEPS

Transitional Probability	North (Negative) Distance Travelled cms	East (Negative) Distance Travelled cms	Sequence of Hourly States
0.036705	- 18,000	0.0	1/1/1/1/1/1/
0.014117	- 16,945	-2,545	1/1/1/1/1/2/
0.00847	- 14,400	-3,600	1/1/1/1/1/3/
		etc.	

It is now possible to obtain the weighted mean distance travelled in the four primary directions using the transitional probabilities as weighting factors. The five hour distance travelled in each of the four directions by months and meters appear in Figures 10 to 13. The total five hour probability for movement in the four major compass points was obtained by developing the fifth (5 hour) order transitional probability mattrix $T^{(5)}_{mn}$ then evaluating (Bharucha-Ried (1)).

$$q^{(5)}_{(n)} = \sum_{m=0}^{\infty} q_{(m)} \; T^{(5)}_{mn} \qquad (2)$$

where $q^{(5)}_{(n)}$ = final probability after 5 hours state

$q_{(n)}$ = iinitial state probability

The final five hour state probability was then summed for the four major compass directions. The transition probabilities for each sequence

were also used as weighting factors to determine the weighted standard deviation for each direction. The standard deviation $\left(\sqrt{\bar{y}^2}\right)$ of the sequence of paths in each direction represent the mean five hour dispersion characteristics in each direction or the mean spread distance of two particles which were released together at the meter after five hours. However, as it was necessary to reduce the number of direction states for computer compatibility, it was not possible to obtain the two-dimensional standard deviations for the four compass directions. The standard deviations obtained for each direction were assumed to be the same for both dimensions (one-dimensional case). From the standard deviations, the five-hour dispersion coefficients were developed from the following equations (Hinze (1)):

$$\bar{y}^2 = 2\epsilon t \tag{3}$$

This equation is limited to long term dispersion or distances of the order of 10^7 cm. The short term form of the equation

$$\bar{y}^2 = \frac{\bar{u}_2^2 \; x_1^2}{\bar{U}_1^2} \tag{4}$$

is restricted to distances less than 10^2cm. The five hour distance considered here varies from 7×10^3 to 5.8×10^4 cm which is in the intermediate range between the short and long term forms of the equations. The dispersion coefficients and the turbulence intensities obtained from equations (2) and (3) for the four directions at Meter 022 in September are presented in Table 6.

TABLE 6

METER 022 SEPTEMBER 1968

Direction	$\sqrt{\overline{y}^2}$ weighted cm	ϵ $cm^2/sec.$	$\overline{U}$ cm/sec.
North	3.92×10^4	4.0×10^4	1.32
East	6.13×10^4	10.5×10^4	4.90
South	3.74×10^4	3.9×10^4	2.14
West	6.52×10^4	12.0×10^4	5.62

It was felt that the long term form of the equation should be utilized as the more conservative estimate of dispersion. A. M. Alsaffar (5) compiled ocean diffusion measurements conducted by thirteen oceanographers and developed a best fit relationship between the diffusion coefficients and spread. Most of the results are contained within an upper limit of

$$\epsilon = 0.09 \left(\sqrt{\overline{y}^2}\right)^{4/3} \qquad (5)$$

and the lower limit

$$\epsilon = 0.04 \left(\sqrt{\overline{y}^2}\right)^{4/3} \qquad (6)$$

The ϵ results obtained in Table 6 are compared with Alsaffar's upper and lower limits, and other Great Lakes measurements in Table 7.

TABLE 7 a

DISPERSION COEFFICIENTS
METER 022 SEPTEMBER

Direction	ϵ cm^2/sec	Alsaffar ϵ_{min} cm^2/sec	Alsaffar ϵ_{max} cm^2/sec	Palmer ϵ $cm^2/sec.$
North	4.0×10^4	2.56×10^4	5.75×10^4	1.28×10^4
East	10.5×10^4	4.48×10^4	10.1×10^4	72.0×10^4
South	3.9×10^4	2.32×10^4	5.23×10^4	1.28×10^4
West	12.0×10^4	4.84×10^4	11.0×10^4	72.0×10^4

TABLE 7 b

DISPERSION COEFFICIENTS
GREAT LAKES STUDIES

		ϵ $cm^2/sec.$
Csanady	(4)	0.04×10^4
Noble	(5)	3 to 6×10^4
Okubo	(6)	0.06×10^4

The dispersion coefficients obtained by this method compare favourably with Alsaffar ocean limits and other Great Lakes Studies. Further, the dispersion coefficients obtained here are an order of magnitude less than the long term coefficients. This is expected on the lakes as the

short term period does not include the full effect of large scale occurrences such as wind seiches which would contribute to the dispersion characteristics.

To obtain measures of the dilution characteristics in each direction after five hours, it is necessary to incorporate the dispersion coefficient in an appropriate dispersion equation. Many of these equations are not suitable as the terms cannot be evaluated from the information obtained from a Markov process (limited due to core space and computational time). Batchelor (7) proposed a one-dimensional diffusion model from a continuous point source.

$$\bar{C}_{max.\,(x)} = \frac{Q}{\sqrt{2\pi}\,(\bar{y}^2)^{1/2}\,U} \qquad \text{.......} (7)$$

$\bar{C}_{max.\,(x)}$ = Maximum concentration mg. / L

Q = Mass discharge mg. / sec.

$\bar{y}^2$ = Weighted mean spread cm.^{2}

U = Weighted mean velocity cm. / sec.

The dilution occurring in each direction after five hours by applying equation (7) is presented in Table 8 .

TABLE 8

FIVE HOUR DILUTION RATES BY DIRECTION
SEPTEMBER METER 022

Direction	Discharge = 78,000 $cm^3/sec.$ max/ per cent
North	100
East	7.2
South	38.0
West	5.5

The flow in equation (7) is selected to illustrate the relative differences of short term dilution rates for each direction.

Results

The results for the short term five hour mean variances (particle spread) are presented in Table 9.

TABLE 9

FIVE HOUR MEAN STANDARD DEVIATIONS

	$\sqrt{\overline{y^2}}$ cm			
Location	South	West	North	East
Meter 022				
September	3.74×10^4	6.52×10^4	3.92×10^4	6.13×10^4
October	6.10×10^4	9.9×10^4	4.1×10^4	9.7×10^4
Meter 023				
September	1.94×10^4	8.34×10^4	5.14×10^4	6.58×10^4
October	7.50×10^4	11.6×10^4	7.0×10^4	9.23×10^4

and the mean five hour velocities are presented in Table 10.

TABLE 10

FIVE HOUR MEAN VELOCITIES

Location	$\overline{U}$ cm/sec. South	West	North	East
Meter 022				
September	2.14	5.62	1.32	4.90
October	3.05	8.00	1.00	7.00
Meter 023				
September	0.93	5.30	1.50	4.30
October	4.28	5.60	2.00	8.80

The dilution characteristics by direction for various flows with the associated five hour probabilities appear in Figures 10 to 13 on pages 18-21.

Conclusion

The short term dispersion method outlined provides a means of determining particle movements and mean dispersion characteristics from current meter records. The high degree of variability of the water movements when considered over short periods of time necessitates the probability and mean five hour approach. It is now possible to compare the short term dispersion patterns for each direction. The method further provides an indication of the maximum distance likely to be travelled in any five hour period.

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NOTATION

M_{ij}	=	one hour transitional state probability matrix for current magnitude " " is initial state " " is final state
D_{KL}	=	one hour transitional state probability matrix for current direction " " is initial state " " is final state
T_{mn}	=	one hour transitional state probability matrix for all current magnitude and direction combinations " " is initial state " " is final state
$T^{(5)}_{mn}$	=	five hour transitional state probability matrix
$q_{(m)}$	=	initial state probability vector
$q^{(5)}_{(n)}$	=	final after five hour state probability vector
$\overline{C}_{max.\ (x)}$	=	mean maximum concentration mg/l, original concentration
ϵ	=	diffusion coefficient in cm^2/sec.
Q	=	discharge in mg/sec
t	=	time in sec.
$\overline{u}^{(2)}_{(1)}, \overline{u}^{(2)}_{(2)}, \overline{u}^{(2)}_{(3)}$	=	fluctuating velocity squared $(cm/sec)^2$
$\overline{U}_{(1)}, \overline{U}_{(2)}, \overline{U}_{(3)}$	=	mean velocity cm/sec.
x_1, x_2, x_3	=	principle axis directions in cm
$\overline{y}^{(2)}$	=	variance of particle separation in cm^2

All directions are coming from for currents.

Date Due
